

Umklapp Contribution to Bulk Absorption in Superconductors

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The second-order umklapp contribution to the bulk absorption of radiation is calculated for superconductors at very low temperatures and is compared with the previously calculated normal phonon contributions. The umklapp absorption spectrum exhibits a sharp peak at the frequency ω_g representing the superconducting energy gap. For Pb, it is not only larger than the normal phonon absorption by many orders of magnitude but is also comparable with the skin absorption. It can also exhibit anisotropy in a single crystal through the polarization factor $(\hat{\epsilon} \cdot \mathbf{g})^2$ (where $\hat{\epsilon}$ is a unit vector in the direction of polarization of the incident radiation and \mathbf{g} = reciprocal lattice vector), which appears in the expression for the umklapp absorption rate.

IN a previous paper,¹ it was shown that the absorption spectrum of a superconductor can have a maximum or "spike" at $\omega < \omega_g$ (= gap frequency) attributed to the *bulk*, phonon processes. While this seemed to agree with the observations by Richards and Tinkham² and Ginsberg and Tinkham³ on Pb and Hg insofar as the presence of the absorption maximum at $\omega < \omega_g$, the theoretical value of the bulk absorption was, however, several orders of magnitude smaller than typical observed value of the *total* absorption. Further, the absorption spectra of superconducting Sn observed by Richards⁴ suggested absorption maxima above the gap frequency ω_g .

In the previous calculations of the bulk absorption of radiation¹ only the *normal* phonon processes were considered; the contribution of the umklapp processes were ignored. The qualitative and quantitative significance of the umklapp processes can, however, vary greatly depending on the temperature range as well as on the materials under consideration. In fact, at very low temperatures where the low-energy phonons are predominant, the momentum transfer $\mathbf{K} = (\mathbf{g} + \mathbf{q}) = (\mathbf{k}_f - \mathbf{k}_i)$ between the final-state \mathbf{k}_f and the initial state \mathbf{k}_i of an electron may be very nearly provided by the reciprocal lattice vector \mathbf{g} alone so that the low-temperature umklapp contribution can be quite substantial for some metals.

In this paper, we present the calculations of the umklapp contribution to the bulk absorption (which we denote as $A^U(\omega)$ / cm-erg) as compared with $A^{NU}(\omega)$ of the *nonumklapp* processes of the previous

calculation): the total *bulk* absorption is given

$$A(\omega) = (A^{NU}(\omega) + A^U(\omega)). \quad (1)$$

The present calculations show that the *bulk* absorption in superconducting Pb may be mostly attributed to the umklapp processes; that the umklapp absorption spectrum has a peak very near the gap frequency ω_g ; and that, due to the umklapp contributions, the peak value of the *bulk* absorption at $\omega \approx \omega_g$ can be comparable, in magnitude, with the *skin* absorption.

Richards' observations⁴ on Sn are also discussed in the light of the present calculations. It is pointed out that the observed anisotropy in the absorption spectra in Sn may be attributed to the polarization factor $(\hat{\epsilon} \cdot \mathbf{g})^2$ ($\hat{\epsilon}$ = unit vector in the direction of the polarization of radiation; \mathbf{g} = reciprocal lattice vector) which appears in the expression for the umklapp absorption rate.

For the umklapp processes, the electron-phonon matrix element $U_{\lambda}^{(\pm)}(\mathbf{k}_f, \mathbf{k}_i)$ may be taken as

$$|U_{\lambda}^{(\pm)}(\mathbf{k}_f, \mathbf{k}_i)|^2 = (1/\rho_M)(\hat{\epsilon}_{\lambda, \mathbf{q}} \cdot \mathbf{K})^2 I^2(\mathbf{K}), \quad (2)$$

$$\mathbf{K} = (\mathbf{q} + \mathbf{g}),$$

where we have replaced $(q^2 C_{\lambda}^2)$ of Ref. 1 by $(\hat{\epsilon}_{\lambda, \mathbf{q}} \cdot \mathbf{K})^2 I^2(\mathbf{K})$, and $I(\mathbf{K})$ (erg) is the strength of the electron-phonon interaction, which may include the "screening" of Bardeen and Pines⁵; $I(\mathbf{K})$ is, in general, weakly dependent on the momenta.⁶

The expression for the total rate of absorption for the second-order processes considered in Ref. 1 now becomes

$$P^U(\text{sec}^{-1}) = \frac{2\pi(\hbar e^2)}{\hbar(\omega V)} \left(\frac{\hbar}{2V\rho_M} \right) \left(\frac{\hbar}{m^*} \right)^2 \left(\frac{1}{\hbar\omega} \right)^2 \sum_{\lambda} \sum_{\mathbf{g}} \frac{4V^2}{(2\pi)^6} \int \dots \int \frac{dS_i}{\hbar v_i} \frac{dS_f}{\hbar v_f} \frac{\rho(E_i)\rho(E_f)}{[N(0)]^2} dE_i dE_f \frac{1}{\omega_{\lambda \mathbf{q}}} (\hat{\epsilon}_{\lambda \mathbf{q}} \cdot \mathbf{K})^2$$

$$\times (\mathbf{K} \cdot \hat{\epsilon})^2 I^2(\mathbf{K}) F(E_i) [1 - F(E_f)] \{ N_{\lambda \mathbf{q}} \delta(E_f - E_i - \hbar\omega_{\lambda \mathbf{q}} - \hbar\omega) + (N_{\lambda \mathbf{q}} + 1) \delta(E_f - E_i + \hbar\omega_{\lambda \mathbf{q}} - \hbar\omega) \}. \quad (3)$$

(Throughout this paper, we use the same notations and approximations as in Ref. 1 unless specified otherwise.)

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¹ E. E. H. Shin, Phys. Rev. **135**, A299 (1964).

² P. L. Richards and M. Tinkham, Phys. Rev. **119**, 575 (1960).

³ D. M. Ginsberg and M. Tinkham, Phys. Rev. **118**, 990 (1960).

⁴ P. L. Richards, Phys. Rev. Letters **7**, 412 (1961).

⁵ J. Bardeen and D. Pines, Phys. Rev. **99**, 1140 (1955).

⁶ See, for example, J. M. Ziman, *Electrons and Phonons* (Oxford University Press, New York, 1960).

As before, we shall consider the situation at very low temperatures such that $\beta\Delta_F, \beta\hbar\omega \gg 1$. At very low temperatures, the statistical mechanical factor (i.e., $N_{\lambda q}$) in the integrand favors the low-energy phonons or low values of q so that

$$\begin{aligned} K &\approx g, \\ q^2 &\ll g^2. \end{aligned} \quad (4)$$

Further, the polarization factors $(\hat{e}_\lambda \cdot \mathbf{K})^2$ and $(\hat{e} \cdot \mathbf{K})^2$ may be replaced by appropriate angular averages. Averaging over the isotropic orientation of \mathbf{q} and neglecting q^2 compared with g^2 , we may take

$$\langle (\hat{e}_\lambda \cdot \mathbf{K})^2 \rangle \approx (\hat{e}_\lambda \cdot \mathbf{g})^2, \quad (5)$$

$$\langle (\hat{e} \cdot \mathbf{K})^2 \rangle \approx (\hat{e} \cdot \mathbf{g})^2, \quad (6)$$

and these out of the momentum integrals. [Note that the approximations of (5) and (6) are not strictly valid in a single crystal, since despite the small magnitude of q compared with \mathbf{g} , $(\hat{e} \cdot \mathbf{q})^2$ can in principle be greater than $(\hat{e} \cdot \mathbf{g})^2$ such as when $\hat{e} \perp \mathbf{g}$. However, for the case of $\langle q^2 \rangle \ll g^2$, (5) and (6) would constitute good approximations except for the very rare situation of having $(\hat{e} \cdot \mathbf{g}) \approx 0$. For a polycrystalline sample, these would be good for all cases at very low temperatures.]

The two surface integrals can be reduced to simpler integrals over q and K in the case of a spherical Fermi surface. By the usual procedure,⁶ the surface integrals are approximated as follows:

$$\begin{aligned} \int \dots \int \frac{dS_i dS_f}{\hbar v_i \hbar v_f} \dots &\approx \frac{1}{\hbar^2 v_F^2} \left(\frac{4\pi^2 k_F^2}{g} \right) \\ &\times \int_0^{q_{\max}} q dq \int_{g-q}^{2k_F} dK \dots, \quad (7) \end{aligned}$$

for the case of $g < 2k_F$, where q_{\max} is equal to the Debye value q_D . On the other hand, if $g > 2k_F$, (7) must be replaced by

$$\frac{1}{\hbar^2 v_F^2} \left(\frac{4\pi^2 k_F^2}{g} \right) \int_{g-2k_F}^{q_{\max}} q dq \int_{g-q}^{2k_F} dK \dots, \quad (8)$$

where the lower limit of the q integral is taken to be $(g - 2k_F) > 0$ to make $K \leq 2k_F$. For the case of $g > 2k_F$, it may be shown that the over-all magnitude of P^U computed from (8) is smaller than that computed from (7) (i.e., for $g < 2k_F$) by a factor which is proportional to

$$\exp[-\beta\hbar S_\lambda(g - 2k_F)].$$

This factor is attributed to the fact that the lower limit of the q integral is $(g - 2k_F) > 0$, and will clearly cut down the contribution of $g > 2k_F$ drastically relative to that of $g < 2k_F$ when the temperature is so low as to make $\beta\hbar S_\lambda(g - 2k_F)$ substantially greater than unity. For this reason, we shall consider only the contribution of $g < 2k_F$ in the calculations for Pb that will follow. (Although contribution of $g > 2k_F$ is small compared with that of $g < 2k_F$, it could, however, be greater than the contribution of the normal phonon processes. This term would then be important for metals with $g > 2k_F$.)⁷

Using (4)–(7) in (3) and completing some algebraic steps, we now obtain the following expressions for $P_{<}^U$ and $P_{>}^U$ associated with $\hbar\omega < 2\Delta_F$ and $\hbar\omega > 2\Delta_F$, respectively;

$$\begin{aligned} \left(\frac{P_{<}^U}{P_{>}^U} \right) (\text{sec}^{-1}) &\approx \frac{1}{\pi^3 m^{*2} v_F^2 \rho_M} \frac{e^2 k_F^2 \hbar \beta}{g \mu^3 S_\lambda^2} (2k_F - g) \left(\frac{J_{<}(\mu)}{J_{>}(\mu)} \right), \quad (9) \end{aligned}$$

where

$$\begin{aligned} J_{<}(\mu) &= \int_{2\beta\Delta_F - \mu}^{\infty} dx \frac{\varphi(x)}{e^x - 1}, \\ J_{>}(\mu) &= J(2\beta\Delta_F) + \int_0^{\mu - 2\beta\Delta_F} dx \frac{\varphi(-x)}{1 - e^{-x}}, \quad (10) \end{aligned}$$

$$J(2\beta\Delta_F) = J_{<}(\mu) \Big|_{\mu=2\beta\Delta_F},$$

and

$$\varphi(x) \approx \frac{2\pi}{2\beta\Delta_F + \mu + x} \left(\frac{\mu + x}{2} \right)^2, \quad (11)$$

up to a multiplicative factor $f(x) \approx O(1)$ as was discussed in Ref. 1, and $\mu = \beta\hbar\omega$.

In (10), $J_{<}(\mu)$ is divergent at $\mu = 2\beta\Delta_F$ while, in $J_{>}(\mu)$, the second term is divergent for $\mu > 2\beta\Delta_F$ and so is $J(2\beta\Delta_F)$. In order to avoid this divergence, we may introduce a cutoff value $x = x_c$ such that $x_c \ll 2\beta\Delta_F$. Introduction of the cutoff x_c is equivalent to making the BCS density-of-states function $\rho(E)$ to be finite at $E = \pm\Delta_F$ such that $\rho(\pm\Delta_F)$ has an effective width of $\Delta E = x_c$; i.e., $\rho(E)$ is cut off at $E = \pm\Delta_F$ with an effective width $\Delta E = x_c \ll \Delta_F$. We may then replace the lower limit of the integral $J_{<}(\mu)$ by $(2\beta\Delta_F - \mu + x_c)$ and the upper and lower limits of the second integral in $J_{>}(\mu)$ by $(\mu - 2\beta\Delta_F + x_c)$ and x_c , respectively. For an order-of-magnitude estimate of the umklapp contribution, we shall use the following approximations for the integrals of $J_{<}(\mu)$ and $J_{>}(\mu)$:

$$\begin{aligned} J_{<}(\mu) &\approx \frac{\pi}{2} (\beta\Delta_F) \int_{2\beta\Delta_F - \mu + x_c}^{\infty} dx \frac{1}{e^x - 1} = \frac{\pi}{2} (\beta\Delta_F) \ln \{ 1 / [1 - e^{-(2\beta\Delta_F - \mu + x_c)}] \} \\ &\approx \frac{\pi}{2} (\beta\Delta_F) e^{-(2\beta\Delta_F - \mu + x_c)} \{ 1 + O(e^{-(2\beta\Delta_F - \mu + x_c)}) \}, \quad (x_c \gtrsim 0) \quad (12) \end{aligned}$$

$$\begin{aligned} J_{>}(\mu) &\approx J(2\beta\Delta_F) + \frac{\pi}{2} \left(\frac{\mu}{2\beta\Delta_F + \mu} \right) \int_{x_c}^{\mu - 2\beta\Delta_F + x_c} dx \frac{1}{1 - e^{-x}} \approx \left\{ \frac{\pi}{2} (\beta\Delta_F) e^{-x_c} + \frac{\pi}{2} \left(\frac{\mu - 2\beta\Delta_F}{\mu + 2\beta\Delta_F} \right) \right\} [1 + O(e^{-x_c})]. \quad (x_c \gtrsim 0) \quad (13) \end{aligned}$$

⁷ The author is indebted to Dr. J. Zak for pointing this out.

[It may be noted that the cutoff width ΔE of the density-of-states function⁸ may not be much more than several percent of the energy gap $2\Delta_F$ so that, for all practical purposes, we may simply take $e^{-x_c} \approx 1$ for μ sufficiently far from $2\beta\Delta_F$ without seriously affecting the orders of magnitude of (12) and (13).]

In (9), the summations over λ and \mathbf{g} factor out completely. It is therefore convenient to define the dimensionless quantity G given by

$$G = \sum_{\lambda} \sum_{\mathbf{g} (< 2k_F)} \left(\frac{v_F}{S_{\lambda}} \right)^2 \frac{(\hat{\epsilon} \cdot \mathbf{g})^2 (\hat{\epsilon}_{\lambda} \cdot \mathbf{g})^2}{g \rho_M} M \left(1 - \frac{g}{2k_F} \right) \times \left(\frac{I(\mathbf{g})}{E_F} \right)^2, \quad (14)$$

and write

$$\begin{aligned} \left(\frac{A_{<}^U(\mu)}{A_{>}^U(\mu)} \right) &= \frac{\beta}{c\mu} \left(\frac{P_{<}^U(\mu)}{P_{>}^U(\mu)} \right) \\ &\cong \frac{2}{\pi^3} \left(\frac{e^2}{\hbar c} \right) \frac{\hbar^2 E_F^2 k_F^3 \beta^2}{m^{*2} v_F^4 M \mu^4} G \left(\frac{J_{<}(\mu)}{J_{>}(\mu)} \right) \text{ (cm}^{-1} \text{ erg}^{-1}), \end{aligned} \quad (15)$$

which is the absorption per unit distance per unit incident energy. This is to be added to $A^{NU}(\mu)$ of Ref. 1 to obtain the total bulk absorption. Here, the dimensionless quantity G contains information on the anisotropy of absorption in a single crystal, since it contains $(\hat{\epsilon} \cdot \mathbf{g})^2 = (g_{11})^2$, g_{11} being the component of \mathbf{g} along the direction of polarization of the incident radiation. On the other hand, if the absorber is not a single crystal and is composed, instead, of a large number of microscopic granules of single crystal, $(\hat{\epsilon} \cdot \mathbf{g})^2$ in G may be replaced by appropriate average value; e.g., $(\hat{\epsilon} \cdot \mathbf{g})^2 \approx \frac{1}{3} g^2$.

We shall now find it interesting to check the magnitude of the umklapp contribution $A^U(\mu)$ to the bulk absorption of the superconducting state as compared with $A^{NU}(\mu)$, which was estimated in Ref. 1. Since we are interested primarily in a small frequency range near the gap frequency ω_g , comparing A^U with A^{NU} at $\omega \cong \omega_g$ should give us a general idea on the relative significance of the two contributions. At $\omega \cong \omega_g$, Eq. (15) gives us

$$A^U(\omega_g) \approx \frac{1}{16\pi^2} \left(\frac{e^2}{\hbar c} \right) \frac{\hbar^2 E_F^2 k_F^3 \beta^2 e^{-x_c}}{m^{*2} v_F^4 M (\beta\Delta_F)^3} G, \quad (16)$$

while Ref. 1 gives us, for the normal phonon contribution,

$$A^{NU}(\omega_g) \approx \frac{1}{8\pi^2} \left(\frac{e^2}{\hbar c} \right) \frac{k_F^2 (k_B T)^5}{(\hbar m^* v_F)^2 \rho_M \Delta_F^3} \sum_{\lambda} \frac{C_{\lambda}^2}{S_{\lambda}^6}. \quad (17)$$

For Pb, the numerical values of most of the parameters which enter in (16) were given in Ref. 1, except for the value of G . In order to estimate G for polycrystal

Pb, we shall use the following approximations:

$$\begin{aligned} (\hat{\epsilon} \cdot \mathbf{g})^2 &\approx \frac{1}{3} g^2; \quad \sum_{\mathbf{g}} \sum_{\lambda} \frac{(\hat{\epsilon}_{\lambda} \cdot \mathbf{g})^2}{S_{\lambda}^2} \approx \frac{\langle z g^2 \rangle_{\text{av}}}{3} \left(\frac{1}{S_L^2} + \frac{2}{S_T^2} \right), \\ G &\approx \frac{z}{9} \frac{\langle g \rangle_{\text{av}} N}{E_F^2} \left(\frac{\langle I^2 \rangle_{\text{av}} \langle g^2 \rangle_{\text{av}}}{N^2} \right) \\ &\quad \times \left[\left(\frac{v_F}{S_L} \right)^2 + 2 \left(\frac{v_F}{S_T} \right)^2 \right] \left(1 - \frac{\langle g \rangle_{\text{av}}}{2k_F} \right), \end{aligned} \quad (18)$$

where S_L and S_T denote the longitudinal and the transverse phase speeds of sound, respectively, and $N = (\rho_M/M)$ (cm^{-3}) is the number of atoms per unit volume, and $\langle g \rangle_{\text{av}}$ and $\langle g^2 \rangle_{\text{av}}$ represent the average values of z number and g and g^2 , respectively.

To compute G_{Pb} , the summation over $g (< 2k_F)$ must be carried over the eight (1,1,1) g values of $2.20 \times 10^8 \text{ cm}^{-1}$ and the six (2,0,0) values of $2.54 \times 10^8 \text{ cm}^{-1}$, all fourteen (i.e., $z=14$) of which are smaller than $2k_F = 3.14 \times 10^8 \text{ cm}^{-1}$. The value of g which results upon averaging the 14 g values is equal to $2.35 \times 10^8 \text{ cm}^{-1}$. As for $(\langle I^2 \rangle_{\text{av}} \langle g^2 \rangle_{\text{av}} / N^2)$, we may use the value

$$(\langle I^2 \rangle_{\text{av}} \langle g^2 \rangle_{\text{av}} / N^2)_{\text{Pb}} = 6.1 \times 10^{-52} \text{ erg}^2 \text{ cm}^4, \quad (19)$$

which is estimated by Rothwarf and Cohen⁹ from the high-temperature resistivity measurements. We also have $N = 3.3 \times 10^{22} \text{ (cm}^{-3}\text{)}$, $S_L = 2.35 \times 10^5 \text{ (cm/sec)}$ and $S_T = 1.27 \times 10^5 \text{ (cm/sec)}$. This gives us

$$G_{\text{Pb}} \cong 2.8 \times 10^{-31} (v_F/E_F)^2. \quad (20)$$

Using this in (16), we obtain

$$A^U(\omega_g) \approx 2 \times 10^{20} e^{-x_c} \text{ cm}^{-1} \text{ erg}^{-1} \quad (21)$$

at $T = 1.4^\circ \text{K}$, where x_c still remains as a parameter. This is to be compared with $A^{NU}(\omega_g) \approx 2 \times 10^9 \text{ cm}^{-1} \text{ erg}^{-1}$ or $A^{NU}(\omega_s) \approx 1 \times 10^{13} \text{ cm}^{-1} \text{ erg}^{-1}$ ($\omega_s =$ the spike frequency) of Ref. 1 at the same temperature. It is seen that, even if we allowed a factor ≈ 10 for e^{-x_c} , $A^U(\omega_g)$ of (21) is many orders of magnitude greater than both $A^{NU}(\omega_g)$ and $A^{NU}(\omega_s)$. [It may be recalled that the normal phonon spectrum has a minimum at $\omega \approx \omega_g$ while here $A^U(\omega)$ has a maximum at $\omega \approx \omega_g$.] It may be shown that an absorption $\approx (10^{19} - 10^{20}) \text{ cm}^{-1} \text{ erg}^{-1}$ corresponds to an absorption coefficient

$$\kappa = 10^4 - 10^5 \text{ cm}^{-1},$$

which compares rather favorably with the penetration depth $\delta \approx 400 \text{ \AA}$ in normal Pb. We therefore conclude that, in Pb and perhaps in other superconductors as well, the bulk absorption at very low temperatures may be attributed mostly to the umklapp processes and may also be comparable, in magnitude, with the absorption due to the skin effects.

⁸ L. C. Hebel, Phys. Rev. **116**, 79 (1959).

⁹ A. Rothwarf and M. Cohen, Phys. Rev. **130**, 1401 (1963).

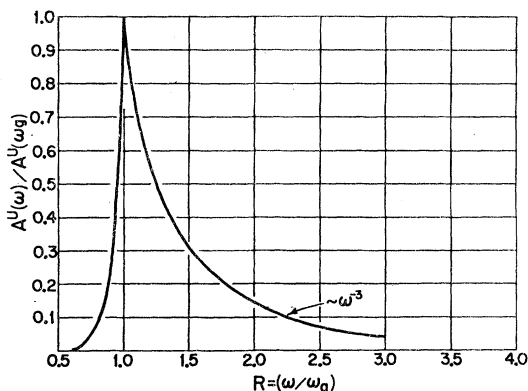


FIG. 1. The umklapp absorption spectrum for Pb at $T=1.4^\circ\text{K}$. $A^U(\omega_g) \approx 10^{19}\text{--}10^{20} \text{ cm}^{-1} \text{ erg}^{-1}$. At $\omega \gtrsim \omega_g$, $A^U(\omega) \propto T$ ($^\circ\text{K}$).

The spectral shape of the umklapp absorption is given by the following relations which are obtained from (15):

$$(A_{<}^U(\omega)/A^U(\omega_g)) \approx (\omega_g/\omega)^4 e^{-\beta(\hbar\omega_g - \hbar\omega)} \quad (22)$$

$$(A_{>}^U(\omega)/A^U(\omega_g)) \approx (\omega_g/\omega)^4 \times \{1 + 2(\omega/\omega_g)((\omega - \omega_g)/(\omega + \omega_g))\}. \quad (23)$$

Note that these ratios depend only on ω_g and (ω/ω_g) at a given temperature so that the shape of the umklapp spectrum would be very nearly the same for different materials to which the present calculations apply, and especially that the spectral shape for $\omega > \omega_g$ is independent of temperature. Equations (22) and (23) are plotted in Fig. 1 for Pb at 1.4°K . At $\omega < \omega_g$, the umklapp absorption decreases very rapidly like $\sim e^{-\beta\hbar(\omega_g - \omega)}$ as shown in (22). At $\omega \gg \omega_g$, it decreases like ω^{-3} and should behave very much like the umklapp absorption in the normal state. It is clear that, if observed, the umklapp absorption would appear mostly as a peak or a spike at $\omega \approx \omega_g$ superimposed on the general background of the skin absorption curve. At other frequencies, it may not be quantitatively significant compared with the skin effect contribution. Formally, the total absorption is given by the bulk absorption plus the skin effect absorption such as what results in the calculations of Mattis and Bardeen¹⁰ and Miller.¹¹ Since the normal phonon contribution to the *bulk* absorption is by many orders of magnitude smaller than the umklapp contribu-

tion, the total (bulk+skin) absorption would be given very nearly by $(A^U(\omega) + A^{\text{skin}}(\omega))$ alone. It may be pointed out further that the absorption maximum at $\omega \approx \omega_g$ in the umklapp spectrum is quite distinct from the one which was obtained in Ref. 1. Previously, the absorption peaks were shifted by noticeable magnitudes below ω_g while, in the umklapp spectrum, the peak appears very nearly at ω_g . If observable, we should therefore see two peaks, one at $\omega < \omega_g$ attributed to the normal phonon processes of Ref. 1 and another at $\omega \approx \omega_g$ attributed to the umklapp processes. In practice, however, we may at best see only the umklapp peak at $\omega \approx \omega_g$ due to the very small magnitude of the normal phonon contribution A^{NU} at very low temperatures. It may be concluded therefore that the observed bumps^{2,3} in the absorption spectra of Pb and Hg (which were previously¹ attributed to the normal phonon contributions) may actually be the umklapp peaks such as shown in Fig. 1. The relative insensitivity of A^U to temperature compared with A^{NU} also supports this conclusion; at $\omega \gtrsim \omega_g$, A^U is linearly dependent on T while $A^{\text{NU}}(\omega_g)$ has a T^5 dependence. Further, the shape of A^U is independent of temperature at $\omega \gtrsim \omega_g$.

It may be pointed out that, although the umklapp spectrum by itself has a peak at $\omega \approx \omega_g$, this peak could appear as a shoulder which is displaced slightly above ω_g in the total (bulk+skin) spectrum if the skin absorption increases faster than the rate at which the umklapp absorption decreases with increase in ω at $\omega \gtrsim \omega_g$. This probably explains the observed absorption maxima⁴ in Sn appearing at $\omega > \omega_g$. If this suggestion on Sn were correct, at least a partial answer to the observed anisotropy⁴ in the absorption spectra of single crystal Sn would lie in the polarization factor $(\hat{e} \cdot \mathbf{g})^2$ which appears in the present expression for the umklapp absorption $A^U(\omega)$; i.e., in addition to whatever contributions a possible anisotropy^{3,4} in the energy gap and of other mechanisms³ might make.

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¹⁰ D. C. Mattis and J. Bardeen, Phys. Rev. **111**, 412 (1958).

¹¹ P. B. Miller, Phys. Rev. **118**, 928 (1960).